

§ [1]. Riemann theory of integration of a bounded function.

Answer: — Suppose f be a bounded function defined on a closed bounded interval $[a, b] \subseteq \mathbb{R}$.

Let $P = \{x_0, x_1, x_2, \dots, x_n = b\}$ be any partition of $[a, b]$ and let $\delta_r = \text{the length of } 84b\text{-interval } [x_{r-1}, x_r], r = 1, 2, 3, \dots, n$.
since f is bounded in $[a, b]$,

therefore, it is necessarily bounded in each sub-interval.

let l.u.b. (supremum) = M_r

and g.l.b. (infimum) of $f = m_r$ in $[x_{r-1}, x_r]; r = 1, 2, 3, \dots$

Definition of Riemann sum

Now we are ⁱⁿ _{attempting} to form the sums

$$U(P, f) = \sum_{r=1}^n M_r \delta_r$$

$$\text{and } L(P, f) = \sum_{r=1}^n m_r \delta_r.$$

Then $U(P, f)$ is called the upper Riemann sum of function f on $[a, b]$ corresponding to the partition P and $L(P, f)$ is called the lower Riemann sum of f corresponding to the partition P . Obviously these sums depend on the function f and the partition P . Since f is the only function we shall denote $U(P, f)$ and $L(P, f)$ in short by $U(P)$ and $L(P)$ respectively.

Thus, the above sums may be written as

$$U(P) = \sum_{r=1}^n M_r \delta_r$$

$$\& L(P) = \sum_{r=1}^n m_r \delta_r.$$

It is clear that the sum $U(P)$ and $L(P)$ exists for any bounded function and for every partition P .

Hence considering all possible modes of sub-division of $[a, b]$, we get a set U of all upper sums and a set L of all lower sums.

We observe now that the sets U and L are bounded.

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